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Finite-Surface Spline

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I. Introduction

IN aeroelastic analyses by finite-element methods, different discretization procedures are employed to determine the structural and aerodynamic loads. To formulate the equations of motion, it is necessary to represent the equivalent air loads at the structural grid points and to represent the structural deflections at the aerodynamic grid points. Over the years, numerous schemes have been proposed to obtain these necessary transformations.[1-6] However, application of these methods frequently impose some restrictions on the format of the structural data, or the methods pose as inconveniences to the user. For example, Ref. 6 requires that the data be given along spanwise or chordwise lines, but structural layouts seldom have such a pattern. Reference 7 has presented an example illustrating that the infinite-surface spline of Harder and Desmarais[5] provides results within acceptable accuracy and user convenience without restriction. However, subsequent experience with the infinite-surface spline indicates that extrapolations to the edges of the planform from the interior structural grid points do not always appear to be reliable. An alternative approach using a network of rectangular grids and hermitian polynominals in each of the rectangular boxes was proposed in Ref. 8. Since no results are reported in this reference, there is very little to say about the success of this method.

The infinite plate-surface spline is appealing in its simplicity, since it has a closed-form solution. Because of observed limitations of the infinite plate, a finite uniform plate with the planform of the aerodynamic lifting surface suggests itself as an alternative with adequate simplicity and reliability. The present method employs uniform plate elements to represent a given planform by a number of quadrilateral or triangular bending elements. A set of constraint conditions using shape functions that are employed in the determination of the stiffness matrix of the plate element are established such that the deformed plate passes through the given data points. Subsequently, a mapping matrix relating displacements at structural and aerodynamic grid points are derived. This transformation matrix provides a general two-dimensional interpolation scheme not limited to the structural and aerodynamic interface, but also applicable to interpolate any smooth data, such as pressure, temperature, or strains.

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II. Derivation of a Mapping Matrix

Consider m number of aerodynamic points at which the displacements and slopes are required in terms of structural displacements given at n points. To achieve this, a linear mapping matrix is developed employing the structural finite-element method, based on the minimum energy principle. In other words, the structure deforms to a unique surface that conforms to the given data points. Figure 1 shows a typical arrangement of a finite-element model of a given planform, as well as representative structural and aerodynamic data points, which may not have a regular arrangement or layout. The displacement w, in the z direction, and rotations (θ about the x axis, ϕ about the y axis) at any point (x,y) within an element, using the notations of Refs. 9 and 10, are given by

$$r = \Omega \rho \tag{1}$$

where

$$r = \begin{Bmatrix} w \\ \theta \\ \phi \end{Bmatrix} \tag{2}$$

$$\Omega = \begin{bmatrix} \omega \\ \omega_y \\ \omega_x \end{bmatrix} \tag{3}$$

$$\rho = \{ w_1\ \theta_1\ \phi_1 \ldots w_4\ \theta_4\ \phi_4 \} \tag{4}$$

in which ω is a (1×12) row matrix of the shape functions (e.g., Ref. 9) used to interpolate the displacements within a four-node quadrilateral element in terms of its nodal degrees of freedom ρ. The vector ρ can be related to the global displacement vector q by means of a Boolean connectivity matrix a.

For example, for the ith element

$$\rho_i = a_i\, q \tag{5}$$

Using Eq. (5) in Eq. (1), the displacement vector r_s for n structural constraint points (subscript s) can be written, after assembly, as

$$q_s = \Psi_s\, q \tag{6}$$

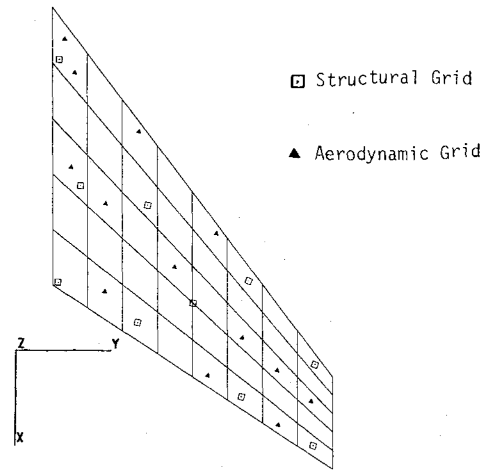

Fig. 1 A typical replacement wing showing arbitrary structural and aerodynamic grids.

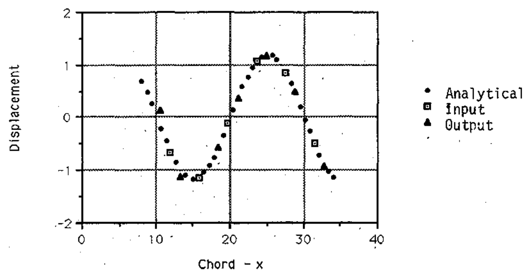

Fig. 2 Deformed wing section showing input and output displacements.

where

$$\Psi = \begin{bmatrix} \Omega_1 a_1 \\ \vdots \\ \Omega_n a_n \end{bmatrix} \tag{7}$$

Similarly, the displacement vector q_a at the aerodynamic points (subscript a), in terms of the global displacement vector q can be written as

$$q_a = \Psi_a q \tag{8}$$

If the given structure is required to pass through a given set of displacements q_s, the equilibrium state of the structure, by the penalty method of constraint (e.g., Ref. 11), is given by

$$[K + \alpha \Psi_s^t \Psi_s] q = \alpha \Psi_s^t q_s \tag{9}$$

where K is the free-free stiffness of the plate and α is a constant.

The constant α can be chosen such that the maximum diagonal element of matrices K and $\Psi_S^t \Psi_S$ are in the same order of magnitude. Note that the constraint matrix is singular. However, when it is added to the singular matrix K, it behaves like a support, rendering the total matrix nonsingular.

Solving for q and substituting into Eq. (8), the displacement vector q_a can be expressed in terms of q_s as follows:

$$q_a = T q_s \tag{10}$$

where

$$T = \Psi_a [(\alpha^{-1}) K + \Psi_s^t \Psi_s]^{-1} \Psi_s^t \tag{11}$$

is the required $(3m \times 3n)$ mapping matrix relating the input function q_s and the output function q_a.

The load transformation matrix from the principal of virtual work[3,6] can be written as

$$R_s = T^t R_a \tag{12}$$

In the present study, a quadrilateral bending element, which employs natural modes proposed by Argyris[9] (C^1-type element) has been utilized. Other types of bending elements (C^0-type element) that employ independent interpolation functions for displacements and rotations (e.g., Ref. 11) were also exercised, but these did not yield satisfactory interpolations. A possible reason for this may be that the stiffness in the w degree of freedom is dominated by the shear strain, which is restricted to certain optimum points within the elements.

The accuracy of the load transformation matrix can be verified by postmultiplying T by a $(3 \times 3n)$ summation matrix $S(x_s, y_s)$ corresponding to the structural points:

$$F = ST \tag{13}$$

where the rows of the ith column of F (say, F_i) denote the sum of the transformed forces and moments due to a unit load at the ith aerodynamic control point. For example, for a unit load at the ith aerodynamic control point (x_a, x_a), the first element of F_i will be unity, and the values of the second and third elements of F_i will be equal to x_a and y_a, respectively, denoting moments due to a unit force. This test failed for C^0 type of bending elements, suggesting that the C^0-type elements do not yield consistent slopes and displacements at all points within an element.

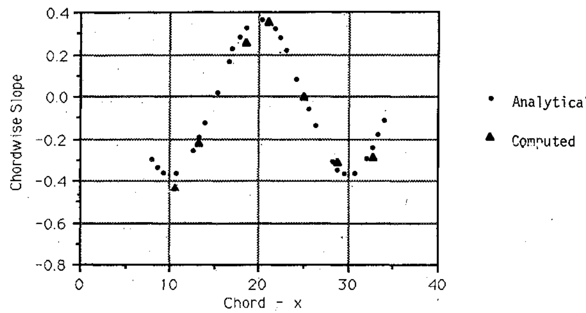

Fig. 3 Comparison of analytical and computed slopes.

III. Results and Conclusions

A typical wing planform shown in Fig. 1 was chosen to indicate the accuracy of the present method. The planform was represented by 40 quadrilateral bending elements. The deformed shape of the structure was represented by the following trignometric function:

$$q_s = A \sin(bx) \cos(cy) \tag{14}$$

where the constants A, b, and c were chosen to represent the deformation having one full wave in the chord direction and one full wave in the span direction. The displacements q_S were calculated at 35 points, along five spanwise strips. The interpolated displacements and slopes were computed at 21 aerodynamic grids using the relation given by Eq. (10).

Figure 2 shows a typical chordwise deformation, in which the square symbols denote the input displacement vector q_s and the triangles denote the interpolate data q_a. The data computed by the trignometric function is also shown by the dotted line. Similarly, the slopes are also shown in Fig. 3. This example demonstrates the accuracy of the transformation matrix T computed by this method. Additional force equilibrium tests according to Eq. (13) have also been conducted with excellent correlation.

Since finite elements are used, any general configuration is amenable to this mapping function, requiring no special arrangement of the input data. This method can also very conveniently be employed to interpolate any two-dimensional continuous function such as pressure, temperature, or strains.

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